

axiomTM



The 30 Year Horizon

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Volume Bibliography: Axiom Literature Citations

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New Foreword

On October 1, 2001 Axiom was withdrawn from the market and ended life as a commercial product. On September 3, 2002 Axiom was released under the Modified BSD license, including this document. On August 27, 2003 Axiom was released as free and open source software available for download from the Free Software Foundation's website, Savannah.

Work on Axiom has had the generous support of the Center for Algorithms and Interactive Scientific Computation (CAISS) at City College of New York. Special thanks go to Dr. Gilbert Baumslag for his support of the long term goal.

The online version of this documentation is roughly 1000 pages. In order to make printed versions we've broken it up into three volumes. The first volume is tutorial in nature. The second volume is for programmers. The third volume is reference material. We've also added a fourth volume for developers. All of these changes represent an experiment in print-on-demand delivery of documentation. Time will tell whether the experiment succeeded.

Axiom has been in existence for over thirty years. It is estimated to contain about three hundred man-years of research and has, as of September 3, 2003, 143 people listed in the credits. All of these people have contributed directly or indirectly to making Axiom available. Axiom is being passed to the next generation. I'm looking forward to future milestones.

With that in mind I've introduced the theme of the "30 year horizon". We must invent the tools that support the Computational Mathematician working 30 years from now. How will research be done when every bit of mathematical knowledge is online and instantly available? What happens when we scale Axiom by a factor of 100, giving us 1.1 million domains? How can we integrate theory with code? How will we integrate theorems and proofs of the mathematics with space-time complexity proofs and running code? What visualization tools are needed? How do we support the conceptual structures and semantics of mathematics in effective ways? How do we support results from the sciences? How do we teach the next generation to be effective Computational Mathematicians?

The "30 year horizon" is much nearer than it appears.

Tim Daly
CAISS, City College of New York
November 10, 2003 ((iHy))

A bibliography of Axiom references which are used throughout Axiom. The first section contains literature that mentions Axiom, initially derived with permission from Nelson Beebe's collection. The second section contains references from Axiom to the literature. The third section sorts papers by topic.

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- introduce better type inference
 - introduce modern language constructs
 - produce understandable diagnostic messages
 - eliminate well known bugs in the type system
 - find new type errors
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Dynamic evaluation is a technique for producing multiple results according to
 a decision tree which evolves with program execution. Sometimes it is desired
 to produce results for all possible branches in the decision tree, while on other
 occasions, it may be sufficient to compute a single result which satisfies certain
 properties. This technique finds use in computer algebra where computing the
 correct result depends on recognizing and properly handling special cases of pa-
 rameters. In previous work, programs using dynamic evaluation have explored all
 branches of decision trees by repeating the computations prior to decision points.

This paper presents two new implementations of dynamic evaluation which avoid recomputing intermediate results. The first approach uses Scheme “continuations” to record state for resuming program execution. The second implementation uses the Unix “fork” operation to form new processes to explore alternative branches in parallel.

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“Type inference in the presence of type abstraction”

ACM SIGPLAN Notices, 24(7) pp192-206 July 1989 CODEN SINODQ ISSN 0362-1340

www.acm.org/pubs/citations/proceedings/pldi/73141/p192-boehm

A number of recent programming language designs incorporate a type checking system based on the Girard-Reynolds polymorphic λ -calculus. This allows the construction of general purpose, reusable software without sacrificing compile-time type checking. A major factor constraining the implementation of these languages is the difficulty of automatically inferring the lengthy type information that is otherwise required if full use is made of these languages. There is no known algorithm to solve any natural and fully general formulation of the “type inference” problem. One very reasonable formulation of the problem is known to be undecidable.

Here we define a restricted version of the type inference problem and present an efficient algorithm for its solution. We argue that the restriction is sufficiently weak to be unobtrusive in practice.

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“Design verification for control engineering”

Proc Fourth International Conference on Integrated Formal Methods, April 2004

We introduce control engineering as a new domain of application for formal methods. We discuss design verification, drawing attention to the role played by diagrammatic evaluation criteria involving numeric plots of a design, such as Nichols and Bode plots. We show that symbolic computation and computational logic can be used to discharge these criteria and provide symbolic, automated, and very general alternatives to these standard numeric tests. We illustrate our work with reference to a standard reference model drawn from military avionics.

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“Etude de la compilation de scratchpad 2”

Rapport de DEA Université de Lille 1, Sept 1991

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“Axiom, language fonctionnel à développement objet”

IT 255, Oct 1993

[Boulanger 93b] Boulanger, Jean-Louis

“AXIOM, A Functional Language with Object Oriented Development”

We present in this paper, a study about the computer algebra system Axiom, which gives us many very interesting Software engineering concepts. This language is a functional language with an Object Oriented Development. This feature is very important for modeling the mathematical world (Hierarchy) and provides a running with mathematical sense. (All objects are functions). We present many problems of running and development in Axiom. We can note that Axiom is the only system of this category.

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“Object Oriented Method for Axiom”

ACM SIGPLAN Notices, 30(2) pp33-41 February 1995 CODEN SINODQ ISSN 0362-1340

Axiom is a very powerful computer algebra system which combines two language paradigms (functional and OOP). Mathematical world is complex and mathematicians use abstraction to design it. This paper presents some aspects of the object oriented development in Axiom. The Axiom programming is based on several new tools for object oriented development, it uses two levels of class and some operations such that *coerce*, *retract*, or *convert* which permit the type evolution. These notions introduce the concept of multi-view.

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We describe an algorithm, based on Risch’s real structure theorem, that determines explicitly all the algebraic relations among a given set of real elementary functions. We also provide examples from its implementation that illustrate the advantages over the use of complex logarithms and exponentials.

[Bronstein 91a] Bronstein, M.

“The Risch differential equation on an algebraic curve”

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“The Risch differential equation on an algebraic curve”

In S.Watt, editor, *Proceedings of ISSAC’91*, pages 241-246, ACM Press, 1991.

[Bronstein 92] Bronstein, M.

“Linear Ordinary Differential Equations: breaking through the order 2 barrier”

www-sop.inria.fr/cafe/Manuel.Bronstein/publications/issac92.ps.gz

A major subproblem for algorithms that either factor ordinary linear differential equations or compute their closed form solutions is to find their solutions y which satisfy $y'/y \in \overline{K}(x)$ where K is the constant field for the coefficients of the

equation. While a decision procedure for this subproblem was known in the 19th century, it requires factoring polynomials over \bar{K} and has not been implemented in full generality. We present here an efficient algorithm for this subproblem, which has been implemented in the AXIOM computer algebra system for equations of arbitrary order over arbitrary fields of characteristic 0. This algorithm never needs to compute with the individual complex singularities of the equation, and algebraic numbers are added only when they appear in the potential solutions. Implementation of the complete Singer algorithm for $n = 2, 3$ based on this building block is in progress.

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“Integration and Differential Equations in Computer Algebra”

We describe in this paper how the problems of computing indefinite integrals and solving linear ordinary differential equations in closed form are now solved by computer algebra systems. After a brief review of the mathematical history of those problems, we outline the two major algorithms for them (respectively the Risch and Singer algorithms) and the recent improvements on those algorithms which has allowed them to be implemented.

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“On Symmetric Powers of Differential Operators”
www-sop.inria.fr/cafe/Manuel.Bronstein/publications/mb_papers.html

We present alternative algorithms for computing symmetric powers of linear ordinary differential operators. Our algorithms are applicable to operators with coefficients in arbitrary integral domains and become faster than the traditional methods for symmetric powers of sufficiently large order, or over sufficiently complicated coefficient domains. The basic ideas are also applicable to other computations involving cyclic vector techniques, such as exterior powers of differential or difference operators.

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 “Domains of data and domains of terms in AXIOM”

 The main new concept we wish to illustrate in this paper is a distinction between
 “domains of data” and “domains of terms”, and its use in the programming of
 certain mathematical structures. Although this distinction is implicit in much of
 the programming work that has gone into the construction of Axiom categories
 and domains, we believe that a formalisation of this is new, that standards and
 conventions are necessary and will be useful in various other contexts. We shall
 show how this concept may be used for the coding of free categories and groupoids
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“Approaching Inheritance from a Natural Mathematical Perspective and from a Java Driven Viewpoint: a Comparative Review”

It is well-known that few object-oriented programming languages allow objects to change their nature at run-time. There have been a number of reasons presented for this, but it appears that there is a real need for matters to change. In this paper we discuss the need for object-oriented programming languages to reflect the dynamic nature of problems, particularly those arising in a mathematical context. It is from this context that we present a framework that realistically represents the dynamic and evolving characteristic of problems and algorithms.

- [Conrad (b)] Conrad, Marc; French, Tim; Maple, Carsten; Pott, Sandra
“Mathematical Use Cases lead naturally to non-standard Inheritance Relationships: How to make them accessible in a mainstream language?”

Conceptually there is a strong correspondence between Mathematical Reasoning and Object-Oriented techniques. We investigate how the ideas of Method Renaming, Dynamic Inheritance and Interclassing can be used to strengthen this relationship. A discussion is initiated concerning the feasibility of each of these features.

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 "The Unknown in Computer Algebra"

Computer algebra systems have to deal with the confusion between "program-
 ming variables" and "mathematical symbols". We claim that they should also
 deal with "unknowns", i.e. elements whose values are unknown, but whose type
 is known. For examples $x^p \neq x$ if x is a symbol, but $x^p = x$ if $x \in GF(p)$. We
 show how we have extended Axiom to deal with this concept.

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Many standard functions, such as the logarithms and square root functions, cannot be defined continuously on the complex plane. Mistaken assumptions about the properties of these functions lead computer algebra systems into various conundrums. We discuss how they can manipulate such functions in a useful fashion.

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[Dos Reis 12] Dos Reis, Gabriel

“A System for Axiomatic Programming”

Proc. Conf. on Intelligent Computer Mathematics, Springer (2012)

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We present the design and implementation of a system for axiomatic programming, and its application to mathematical software construction. Key novelties include a direct support for user-defined axioms establishing local equality between types, and overload resolution based on equational theories and user-defined local axioms. We illustrate uses of axioms, and their organization into concepts, in structured generic programming as practiced in computational mathematical systems.

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“Order Sorted Computer Algebra and Coercions”

Ph.D. Thesis University of Bath 1997

Computer algebra systems are large collections of routines for solving mathematical problems algorithmically, efficiently and above all, symbolically. The more advanced and rigorous computer algebra systems (for example, Axiom) use the concept of strong types based on order-sorted algebra and category theory to ensure that operations are only applied to expressions when they “make sense”.

In cases where Axiom uses notions which are not covered by current mathematics we shall present new mathematics which will allow us to prove that all such cases are reducible to cases covered by the current theory. On the other hand, we shall also point out all the cases where Axiom deviates undesirably from the mathematical ideal. Furthermore we shall propose solutions to these deviations.

Strongly typed systems (especially of mathematics) become unusable unless the system can change the type in a way a user expects. We wish any change expected by a user to be automated, “natural”, and unique. “Coercions” are normally viewed as “natural type changing maps”. This thesis shall rigorously define the word “coercion” in the context of computer algebra systems.

We shall list some assumptions so that we may prove new results so that all coercions are unique. This concept is called “coherence”.

We shall give an algorithm for automatically creating all coercions in type system which adheres to a set of assumptions. We shall prove that this is an algorithm and that it always returns a coercion when one exists. Finally, we present a demonstration implementation of this automated coercion algorithm in Axiom.

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“Modeling Inheritance as Coercion in a Symbolic Computation System” ISSAC 2001 ACM 1-58113-417-7/01/0007

In this paper the analysis of the data structures used in a symbolic computation system, called Kenzo, is undertaken. We deal with the specification of the inheritance relationship since Kenzo is an object-oriented system, written in CLOS, the Common Lisp Object System. We focus on a particular case, namely the relationship between simplicial sets and chain complexes, showing how the order-sorted algebraic specifications formalisms can be adapted, through the “inheritance as coercion” metaphor, in order to model this Kenzo fragment.

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Some of the earliest computer algebra systems (CAS) looked like overloaded languages of the same era. FORMAC, PL/I FORMAC, Formula Algol, and others each took advantage of a pre-existing language base and expanded the notion of a numeric value to include mathematical expressions. Much more recently, perhaps encouraged by the growth in popularity of C++, we have seen a renewal of the use of overloading to implement a CAS.

This paper makes three points. 1. It is easy to do overloading in Common Lisp, and show how to do it in detail. 2. Overloading per se provides an easy solution to some simple programming problems. We show how it can be used for a “demonstration” CAS. Other simple and plausible overloadings interact nicely with this basic system. 3. Not all goes so smoothly: we can view overloading as a case study and perhaps an object lesson since it fails to solve a number of fairly-well articulated and difficult design issues in CAS for which other approaches are preferable.

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“About the Polynomial System Solve Facility of Axiom, Macyma, Maple Mathematica, MuPAD, and Reduce”

We report on some experiences with the general purpose Computer Algebra Systems (CAS) Axiom, Macsyma, Maple, Mathematica, MuPAD, and Reduce solving systems of polynomial equations and the way they present their solutions. This snapshot (taken in the spring of 1996) of the current power of the different systems in a special area concentrates on both CPU-times and the quality of the output.

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The SCRATCHPAD/1 system is designed to provide an interactive symbolic computational facility for the mathematician user. The system features a user language designed to capture the style and succinctness of mathematical notation, together with a facility for conveniently introducing new notations into the language. A comprehensive system library incorporates symbolic capabilities provided by such systems as SIN, MATHLAB, and REDUCE.

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This thesis presents an algorithm for computing (one-sided) limits within a sym-
 bolic manipulation system. Computing limits is an important facility, as limits
 are used both by other functions such as the definite integrator and to get directly
 some qualitative information about a given function.

The algorithm we present is very compact, easy to understand and easy to imple-
 ment. It overcomes the cancellation problem other algorithms suffer from. These
 goals were achieved using a uniform method, namely by expanding the whole
 function into a series in terms of its most rapidly varying subexpression instead
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“Next Generation Computer Algebra Systems AXIOM and the Scratchpad Concept: Applications to Research in Algebra”

21st Nordic Congress of Mathematicians 1992

One way in which mathematicians deal with infinite amounts of data is symbolic representation. A simple example is the quadratic equation

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

a formula which uses symbolic representation to describe the solutions to an infinite class of equations. Most computer algebra systems can deal with polynomials with symbolic coefficients, but what if symbolic exponents are called for (e.g. $1 + t^i$)? What if symbolic limits on summations are also called for, for example

$$1 + t + \dots + t^i = \sum_j t^j$$

The “Scratchpad Concept” is a theoretical ideal which allows the implementation of objects at this level of abstraction and beyond in a mathematically consistent way. The Axiom computer algebra system is an implementation of a major part of the Scratchpad Concept. Axiom (formerly called Scratchpad) is a language with extensible parameterized types and generic operators which is based on the notions of domains and categories. By examining some aspects of the Axiom system, the Scratchpad Concept will be illustrated. It will be shown how some complex problems in homological algebra were solved through the use of this system.

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3rd International Conf. on Expert Systems in Numerical Computing 1993

The Axiom language is based on the notions of “categories”, “domains”, and “packages”. These concepts are used to build an interface between symbolic and numeric calculations. In particular, an interface to the NAG Fortran Library and Axiom’s algebra and graphics facilities is presented. Some examples of numerical calculations in a symbolic computational environment are also included using the finite element method. While the examples are elementary, we believe that they point to very powerful methods for combining numeric and symbolic computational techniques.

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 This paper proposes a non-intrusive automatic parallelization framework for typeful and property-aware computer algebra systems.
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- This paper investigates some soundness conditions which have to be fulfilled in systems with coercions and generic operators. A result of Reynolds on unrestricted generic operators is extended to generic operators which obey certain constraints. We get natural conditions for such operators, which are expressed within the theoretic framework of category theory. However, in the context of computer algebra, there arise examples of coercions and generic operators which do not fulfil these conditions. We describe a framework – relaxing the above conditions – that allows distinguishing between cases of ambiguities which can be resolved in a quite natural sense and those which cannot. An algorithm is presented that detects such unresolvable ambiguities in expressions.
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 “From Untyped to Polymorphically Typed Objects in Mathematical Web Services”

OpenMath is a widely recognized approach to the semantic markup of mathematics that is often used for communication between OpenMath compliant systems. The Aldor language has a sophisticated category-based type system that was specifically developed for the purpose of modelling mathematical structures, while the system itself supports the creation of small-footprint applications suitable for deployment as web services. In this paper we present our first results of how one may perform translations from generic OpenMath objects into values in specific Aldor domains, describing how the Aldor interface domain Expression-Tree is used to achieve this. We outline our Aldor implementation of an OpenMath translator, and describe an efficient extension of this to the Parser category. In addition, the Aldor service creation and invocation mechanism are explained. Thus we are in a position to develop and deploy mathematical web services whose descriptions may be directly derived from Aldor's rich type language.

[Naylor 95] Naylor, Bill

“Symbolic Interface for an advanced hyperbolic PDE solver”

www.sci.csd.uwo.ca/~bill/Papers/symbInterface2.ps

An Axiom front end is described, which is used to generate mathematical objects needed by one of the latest NAG routines, to be included in the Mark 17 version of the NAG Numerical library. This routine uses powerful techniques to find the solution to Hyperbolic Partial Differential Equations in conservation form and in one spatial dimension. These mathematical objects are non-trivial, requiring much mathematical knowledge on the part of the user, which is otherwise irrelevant to the physical problem which is to be solved. We discuss the individual mathematical objects, considering the mathematical theory which is relevant, and some of the problems which have been encountered and solved during the FORTRAN generation necessary to realise the object. Finally we display some of our results.

[Naylor 00b] Naylor, W.A.; Davenport, J.H.

“A Monte-Carlo Extension to a Category-Based Type System”

www.sci.csd.uwo.ca/~bill/Papers/monteCarCat3.ps

The normal claim for mathematics is that all calculations are 100% accurate and therefore one calculation can rely completely on the results of sub-calculations, however there exist *Monte-Carlo* algorithms which are often much faster than the equivalent deterministic ones where the results will have a prescribed probability (presumably small) of being incorrect. However there has been little discussion of how such algorithms can be used as building blocks in Computer Algebra. In this paper we describe how the computational category theory which is the basis of the type structure used in the Axiom computer algebra system may be extended to cover probabilistic algorithms, which use Monte-Carlo techniques. We follow this with a specific example which uses Straight Line Program representation.

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LCCN QA76.5.S94 1971
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“The type system of Axiom”

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“Applications of a software interconnection system in mathematical problem solving environments” In Bruce W. Char, editor. Proceedings of the 1986 Symposium on Symbolic and Algebraic Computation: SYMSAC '86, July 21-23, ACM Press, New York, NY 10036, USA, 1986. ISBN 0-89791-199-7 LCCN QA155.7.E4 A281 1986 ACM order number 505860

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“2014: 30+ Years Common Lisp the Language”
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[Robidoux 93] Robidoux, Nicolas

“Does Axiom Solve Systems of O.D.E’s Like Mathematica?”
 July 1993

If I were demonstrating Axiom and were asked this question, my reply would be “No, but I am not sure that this is a bad thing”. And I would illustrate this with the following example.

Consider the following system of O.D.E.’s

$$\begin{aligned}\frac{dx_1}{dt} &= \left(1 + \frac{\cos t}{2 + \sin t}\right) x_1 \\ \frac{dx_2}{dt} &= x_1 - x_2\end{aligned}$$

This is a very simple system: x_1 is actually uncoupled from x_2

[Rioboo 92] Rioboo, R.

“Real algebraic closure of an ordered field, implementation in Axiom”
 In Wang [Wan92], pp206-215 ISBN 0-89791-489-9 (soft cover) 0-89791-490-2 (hard cover) LCCN QA76.95.I59 1992

Real algebraic numbers appear in many Computer Algebra problems. For instance the determination of a cylindrical algebraic decomposition for an euclidean space requires computing with real algebraic numbers. This paper describes an implementation for computations with the real roots of a polynomial. This process is designed to be recursively used, so the resulting domain of computation is the set of all real algebraic numbers. An implementation for the real algebraic closure has been done in Axiom (previously called Scratchpad).

[Roesner 95] Roesner, K. G.

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 SIGSAM Bulletin (ACM Special Interest Group on Symbolic and Algebraic Manipulation), 25(2) pp4-17 April 1991 CODEN SIGSBZ ISSN 0163-5824
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 “Analysis and Application of the Formal Theory of Partial Differential Equations”
 PhD thesis, School of Physics and Materials, Lancaster University (1994)
www.mathematik.uni-kassel.de/~seiler/Papers/Diss/diss.ps.gz

An introduction to the formal theory of partial differential equations is given emphasizing the properties of involutive symbols and equations. An algorithm to complete any differential equation to an involutive one is presented. For an involutive equation possible values for the number of arbitrary functions in its general solution are determined. The existence and uniqueness of solutions for analytic equations is proven. Applications of these results include an analysis of symmetry and reduction methods and a study of gauge systems. It is shown that the Dirac algorithm for systems with constraints is closely related to the

completion of the equation of motion to an involutive equation. Specific examples treated comprise the Yang-Mills Equations, Einstein Equations, complete and Jacobian systems, and some special models in two and three dimensions. To facilitate the involved tedious computations an environment for geometric approaches to differential equations has been developed in the computer algebra system Axiom. The appendices contain among others brief introductions into Cartan-Kähler Theory and Janet-Riquier Theory.

[Seiler 94a] Seiler, W.M.

“Completion to involution in AXIOM”
in Calmet [Cal94] pp103-104

[Seiler 94b] Seiler, W.M.

“Pseudo differential operators and integrable systems in AXIOM”
Computer Physics Communications, 79(2) pp329-340 April 1994 CODEN CPHCBZ
ISSN 0010-4655

An implementation of the algebra of pseudo differential operators in the computer algebra system Axiom is described. In several examples the application of the package to typical computations in the theory of integrable systems is demonstrated.

[Seiler 95] Seiler, W.M.

“Applying AXIOM to partial differential equations”
Internal Report 95-17, Universität Karlsruhe, Fakultät für Informatik 1995

We present an Axiom environment called JET for geometric computations with partial differential equations within the framework of the jet bundle formalism. This comprises especially the completion of a given differential equation to an involutive one according to the Cartan-Kuranishi Theorem and the setting up of the determining system for the generators of classical and non-classical Lie symmetries. Details of the implementations are described and applications are given. An appendix contains tables of all exported functions.

[Seiler 95b] Seiler, W.M.; Calmet, J.

“JET – An Axiom Environment for Geometric Computations with Differential Equations”

JET is an environment within the computer algebra system Axiom to perform such computations. The current implementation emphasises the two key concepts involution and symmetry. It provides some packages for the completion of a given system of differential equations to an equivalent involutive one based on the Cartan-Kuranishi theorem and for setting up the determining equations for classical and non-classical point symmetries.

[Seiler 97] Seiler, Werner M.

“Computer Algebra and Differential Equations: An Overview”
www.mathematik.uni-kassel.de/~seiler/Papers/Postscript/CADERep.ps.gz

We present an informal overview of a number of approaches to differential equations which are popular in computer algebra. This includes symmetry and completion theory, local analysis, differential ideal and Galois theory, dynamical systems and numerical analysis. A large bibliography is provided.

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 “Algorithmic differentiation in Axiom”
 ACM SIGSAM ISSAC Proceedings 2007 Waterloo, Canada 2007 pp347-354 ISBN 978-1-59593-743-8

This paper describes the design and implementation of an algorithmic differentiation framework in the Axiom computer algebra system. Our implementation works by transformations on Spad programs at the level of the typed abstract syntax tree.

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In Buchberger and Caviness [BC85], pp32-33 ISBN 0-387-15983-5 (vol. 1), 0-387-15984-3 (vol. 2) LCCN QA155.7.E4 E86 1985 Two volumes.

[Sutor 87a] Sutor, R. S.; Jenks, R. D.

“The type inference and coercion facilities in the Scratchpad II interpreter” In Wexelblat [Wex87], pp56-63 ISBN 0-89791-235-7 LCCN QA76.7.S54 v22:7 SIGPLAN Notices, v22 n7 (July 1987)

[Sutor 87b] Sutor, Robert S.

“The Scratchpad II Computer Algebra System. Using and Programming the Interpreter”

IBM Course presentation slide deck Spring 1987

[Sutor 87c] Sutor, Robert S.; Jenks, Richard

“The type inference and coercion facilities in the Scratchpad II interpreter”

Research report RC 12595 (#56575), IBM Thomas J. Watson Research Center, Yorktown Heights, NY, USA, 1987, 11pp

The Scratchpad II system is an abstract datatype programming language, a compiler for the language, a library of packages of polymorphic functions and parameterized abstract datatypes, and an interpreter that provides sophisticated type inference and coercion facilities. Although originally designed for the implementation of symbolic mathematical algorithms, Scratchpad II is a general purpose programming language. This paper discusses aspects of the implementation of the interpreter and how it attempts to provide a user friendly and relatively weakly typed front end for the strongly typed programming language.

[Sutor 88] Sutor, Robert S.

“A guide to programming in the scratchpad 2 interpreter”

IBM Manual, March 1988

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[Thompson 00] Thompson, Simon

“Logic and dependent types in the Aldor Computer Algebra System”

We show how the Aldor type system can represent propositions of first-order logic, by means of the ‘propositions as types’ correspondence. The representation relies on type casts (using `pretend`) but can be viewed as a prototype implementation of a modified type system with *type evaluation* reported elsewhere. The logic is used to provide an axiomatisation of a number of familiar Aldor categories as well as a type of vectors.

[Thompson (a)] Thompson, Simon; Timochouk, Leonid

“The Aldor language”

This paper introduces the `Aldor--` language, which is a functional programming language with dependent types and a powerful, type-based, overloading mechanism. The language is built on a subset of Aldor, the ‘library compiler’ language for the Axiom computer algebra system. `Aldor--` is designed with the intention of incorporating logical reasoning into computer algebra computations.

The paper contains a formal account of the semantics and type system of `Aldor--`; a general discussion of overloading and how the overloading in `Aldor--` fits into the general scheme; examples of logic within `Aldor--` and notes on the implementation of the system.

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 "Type Systems for Computer Algebra"
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 An important feature of modern computer algebra systems is the support of a rich type system with the possibility of type inference. Basic features of such a type system are polymorphism and coercion between types. Recently the use of order-sorted rewrite systems was proposed as a general framework. We will give a quite simple example of a family of types arising in computer algebra whose coercion relations cannot be captured by a finite set of first-order rewrite rules.
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cg.cs.uni-bonn.de/personal-pages/weber/publications/pdf/WeberA/Weber92a.pdf
 Most existing computer algebra systems are pure symbol manipulating systems without language support for the occurring types. This is mainly due to the fact that the occurring types are much more complicated than in traditional programming languages. In the last decade the study of type systems has become an active area of research. We will give a proposal for a type system showing that several problems for a type system of a symbolic computation system can

be solved by using results of this research. We will also provide a variety of examples which will show some of the problems that remain and that will require further research.

[Weber 93b] Weber, Andreas

“Type Systems for Computer Algebra”

cg.cs.uni-bonn.de/personal-pages/weber/publications/pdf/WeberA/Weber93b.pdf

We study type systems for computer algebra systems, which frequently correspond to the “pragmatically developed” typing constructs used in AXIOM. A central concept is that of *type classes* which correspond to AXIOM categories. We will show that types can be syntactically described as terms of a regular order-sorted signature if no type parameters are allowed. Using results obtained for the functional programming language Haskell we will show that the problem of *type inference* is decidable. This result still holds if higher-order functions are present and *parametric polymorphism* is used. These additional typing constructs are useful for further extensions of existing computer algebra systems: These typing concepts can be used to implement category theoretic constructs and there are many well known constructive interactions between category theory and algebra.

[Weber 94] Weber, Andreas

“Algorithms for Type Inference with Coercions”

ISSAC 94 ACM 0-89791-638-7/94/0007

This paper presents algorithms that perform a type inference for a type system occurring in the context of computer algebra. The type system permits various classes of coercions between types and the algorithms are complete for the precisely defined system, which can be seen as a formal description of an important subset of the type system supported by the computer algebra program Axiom.

Previously only algorithms for much more restricted cases of coercions have been described or the frameworks used have been so general that the corresponding type inference problems were known to be undecidable.

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“On coherence in computer algebra”

cg.cs.uni-bonn.de/personal-pages/weber/publications/pdf/WeberA/Weber94e.pdf

Modern computer algebra systems (e.g. AXIOM) support a rich type system including parameterized data types and the possibility of implicit coercions between types. In such a type system it will be frequently the case that there are different ways of building coercions between types. An important requirement is that all coercions between two types coincide, a property which is called *coherence*. We will prove a coherence theorem for a formal type system having several possibilities of coercions covering many important examples. Moreover, we will give some informal reasoning why the formally defined restrictions can be satisfied by an actual system.

[Weber 96] Weber, Andreas

“Computing Radical Expressions for Roots of Unity”

cg.cs.uni-bonn.de/personal-pages/weber/publications/pdf/WeberA/Weber96a.pdf

We present an improvement of an algorithm given by Gauss to compute a radical expression for a p -th root of unity. The time complexity of the algorithm is $O(p^3 m^6 \log p)$, where m is the largest prime factor of $p - 1$.

[Weber 99] Weber, Andreas

“Solving Cyclotomic Polynomials by Radical Expressions”

cg.cs.uni-bonn.de/personal-pages/weber/publications/pdf/WeberA/WeberKeckeisen99a.pdf

We describe a Maple package that allows the solution of cyclotomic polynomials by radical expressions. We provide a function that is an extension of the Maple *solve* command. The major algorithmic ingredient of the package is an improvement of a method due to Gauss which gives radical expressions for roots of unity. We will give a summary for computations up to degree 100, which could be done within a few hours of cpu time on a standard workstation.

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We present a new rational algorithm for solving Risch differential equations in towers of transcendental elementary extensions. In contrast to a recent algorithm by Davenport we do not require a progressive reduction of the denominators involved, but use weak normality to obtain a formula for the denominator of a possible solution. Implementation timings show this approach to be faster than a Hermite-like reduction.

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J. Symbolic Computation (1990) 9, pp117-173 September 1988

We extend a recent algorithm of Trager to a decision procedure for the indefinite integration of elementary functions. We can express the integral as an elementary function or prove that it is not elementary. We show that if the problem of integration in finite terms is solvable on a given elementary function field k , then it is solvable in any algebraic extension of $k(\theta)$, where θ is a logarithm or exponential of an element of k . Our proof considers an element of such an extension field to be an algebraic function of one variable over k .

In his algorithm for the integration of algebraic functions, Trager describes a Hermite-type reduction to reduce the problem to an integrand with only simple finite poles on the associated Riemann surface. We generalize that technique to curves over liouvillian ground fields, and use it to simplify our integrands. Once the multiple finite poles have been removed, we use the Puiseux expansions of the integrand at infinity and a generalization of the residues to compute the integral. We also generalize a result of Rothstein that gives us a necessary condition for elementary integrability, and provide examples of its use.

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The automatic computation of limits can be reduced to two main sub-problems. The first one is asymptotic comparison where one must decide automatically which one of two functions in a specified class dominates the other one asymptotically. The second one is asymptotic cancellation and is usually exemplified by

$$e^x[\exp(1/x + e^{-x}) - \exp(1/x)], \quad x \leftarrow \infty$$

In this example, if the sum is expanded in powers of $1/x$, the expansion always yields $O(x^{-k})$, and this is not enough to conclude.

In 1990, J.Shackell found an algorithm that solved both these problems for the case of $\exp - \log$ functions, i.e. functions build by recursive application of exponential, logarithm, algebraic extension and field operations to one variable and the rational numbers. D. Gruntz and G. Gonnet propose a slightly different algorithm for exp-log functions. Extensions to larger classes of functions are also discussed.

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1.3 Special Topics

Solving Systems of Equations

[Bronstein 86] Bronstein, Manuel

“Gsolve: a faster algorithm for solving systems of algebraic equations”
Proc of 5th ACM SYMSAC (1986) pp247-249 ISBN 0-89791-199-7

We apply the elimination property of Gröbner bases with respect to pure lexicographic ordering to solve systems of algebraic equations. We suggest reasons for this approach to be faster than the resultant technique, and give examples and timings that show that it is indeed faster and more correct, than MACSYMA’s solve.

Numerical Algorithms

[Bronstein 99] Bronstein, Manuel

“Fast Deterministic Computation of Determinants of Dense Matrices”
www-sop.inria.fr/cafe/Manuel.Bronstein/publications/mb_papers.html

In this paper we consider deterministic computation of the exact determinant of a dense matrix M of integers. We present a new algorithm with worst case complexity

$$O(n^4(\log n + \log ||M||) + x^3 \log^2 ||M||)$$

, where n is the dimension of the matrix and $||M||$ is a bound on the entries in M , but with average expected complexity

$$O(n^4 + m^3(\log n + \log ||M||)^2)$$

, assuming some plausible properties about the distribution of M . We will also describe a practical version of the algorithm and include timing data to compare this algorithm with existing ones. Our result does not depend on “fast” integer or matrix techniques.

[Kelsey 00] Kelsey, Tom

“Exact Numerical Computation via Symbolic Computation”
tom.host.cs.st-andrews.ac.uk/pub/ccpaper.pdf

We provide a method for converting any symbolic algebraic expression that can be converted into a floating point number into an exact numeric representation. We use this method to demonstrate a suite of procedures for the representation of, and arithmetic over, exact real numbers in the Maple computer algebra system. Exact reals are represented by potentially infinite lists of binary digits, and interpreted as sums of negative powers of the golden ratio.

[Yang 14] Yang, Xiang; Mittal, Rajat

“Acceleration of the Jacobi iterative method by factors exceeding 100 using scheduled

relation”

engineering.jhu.edu/fsag/wp-content/uploads/sites/23/2013/10/JCP_revised_WebPost.pdf

Special Functions

[Corless 05] Corless, Robert M.; Jeffrey, David J.; Watt, Stephen M.; Bradford, Russell; Davenport, James H.

“Reasoning about the elementary functions of complex analysis”

www.csd.uwo.ca/~watt/pub/reprints/2002-amai-reasoning.pdf

There are many problems with the simplification of elementary functions, particularly over the complex plane. Systems tend to make “howlers” or not to simplify enough. In this paper we outline the “unwinding number” approach to such problems, and show how it can be used to prevent errors and to systematise such simplification, even though we have not yet reduced the simplification process to a complete algorithm. The unsolved problems are probably more amenable to the techniques of artificial intelligence and theorem proving than the original problem of complex-variable analysis.

Exponential Integral $E_1(x)$

[Segletes 98] Segletes, S.B.

“A compact analytical fit to the exponential integral $E_1(x)$ ”

Technical Report ARL-TR-1758, U.S. Army Ballistic Research Laboratory, Aberdeen Proving Ground, MD, September 1998

A four-parameter fit is developed for the class of integrals known as the exponential integral (real branch). Unlike other fits that are piecewise in nature, the current fit to the exponential integral is valid over the complete domain of the function (compact) and is everywhere accurate to within $\pm 0.0052\%$ when evaluating the first exponential integral, E_1 . To achieve this result, a methodology that makes use of analytically known limiting behaviors at either extreme of the domain is employed. Because the fit accurately captures limiting behaviors of the E_1 function, more accuracy is retained when the fit is used as part of the scheme to evaluate higher-order exponential integrals, E_n , as compared with the use of brute-force fits to E_1 , which fail to accurately model limiting behaviors. Furthermore, because the fit is compact, no special accommodations are required (as in the case of spliced piecewise fits) to smooth the value, slope, and higher derivatives in the transition region between two piecewise domains. The general methodology employed to develop this fit is outlined, since it may be used for other problems as well.

[Segletes 09] Segletes, S.B.

“Improved fits for $E_1(x)$ vis-à-vis those presented in ARL-TR-1758

Technical Report ARL-TR-1758, U.S. Army Ballistic Research Laboratory, Aberdeen Proving Ground, MD, September 1998

This is a writeup detailing the more accurate fits to $E_1(x)$, relative to those presented in ARL-TR-1758. My actual fits are to

$$F1 = [x \exp(x) E_1(x)]$$

which spans a functional range from 0 to 1. The best accuracy I have been yet able to achieve, defined by limiting the value of

$$[(F1)_{fit} - F1]/F1$$

over the domain, is approximately 3.1E-07 with a 12-parameter fit, which unfortunately isn't quite to 32-bit floating-point accuracy. Nonetheless, the fit is not a piecewise fit, but rather a single continuous function over the domain of non-negative x , which avoids some of the problems associated with piecewise domain splicing.

Polynomial GCD

[Knuth 71] Knuth, Donald

“The Art of Computer Programming”

2nd edition Vol. 2 (Seminumerical Algorithms) 1st edition, 2nd printing,
Addison-Wesley 1971, section 4.6 pp399-505

[Ma 90] Ma, Keju; Gathen, Joachim von zur

“Analysis of Euclidean Algorithms for Polynomials over Finite Fields”

J. Symbolic Computation (1990) Vol 9 pp429-455

www.researchgate.net/publication/220161718_Analysis_of_Euclidean_Algorithms_for_Polynomials_over_Finite_Fields/file/60b7d52b326a1058e4.pdf

This paper analyzes the Euclidean algorithm and some variants of it for computing the greatest common divisor of two univariate polynomials over a finite field. The minimum, maximum, and average number of arithmetic operations both on polynomials and in the ground field are derived.

[Naylor 00a] Naylor, Bill

“Polynomial GCD Using Straight Line Program Representation”

PhD. Thesis, University of Bath, 2000

www.sci.csd.uwo.ca/~bill/thesis.ps

This thesis is concerned with calculating polynomial greatest common divisors using straight line program representation.

In the Introduction chapter, we introduce the problem and describe some of the traditional representations for polynomials, we then talk about some of the general subjects central to the thesis, terminating with a synopsis of the category theory which is central to the Axiom computer algebra system used during this research.

The second chapter is devoted to describing category theory. We follow with a chapter detailing the important sections of computer code written in order to investigate the straight line program subject. The following chapter on evaluation strategies and algorithms which are dependant on these follows, the major algorithm which is dependant on evaluation and which is central to our thesis being that of equality checking. This is indeed central to many mathematical problems. Interpolation, that is the determination of coefficients of a polynomial is the subject of the next chapter. This is very important for many straight line program algorithms, as their non-canonical structure implies that it is relatively difficult to determine coefficients, these being the basic objects that many algorithms work on. We talk about three separate interpolation techniques and compare their advantages and disadvantages. The final two chapters describe some of the results we have obtained from this research and finally conclusions we have drawn as to the viability of the straight line program approach and possible extensions.

Finally we terminate with a number of appendices discussing side subjects encountered during the thesis.

[Shoup 93] Shoup, Victor

“Factoring Polynomials over Finite Fields: Asymptotic Complexity vs Reality*”

Proc. IMACS Symposium, Lille, France, (1993) www.shoup.net/papers/lille.pdf

This paper compares the algorithms by Berlekamp, Cantor and Zassenhaus, and Gathen and Shoup to conclude that (a) if large polynomials are factored the FFT should be used for polynomial multiplication and division, (b) Gathen and Shoup should be used if the number of irreducible factors of f is small. (c) if nothing is known about the degrees of the factors then Berlekamp’s algorithm should be used

[Gathen 01] Gathen, Joachim von zur; Panario, Daniel

“Factoring Polynomials Over Finite Fields: A Survey”

J. Symbolic Computation (2001) Vol 31, pp3-17

people.csail.mit.edu/dmoshdov/courses/codes/poly-factorization.pdf

This survey reviews several algorithms for the factorization of univariate polynomials over finite fields. We emphasize the main ideas of the methods and provide an up-to-date bibliography of the problem. This paper gives algorithms for *squarefree factorization*, *distinct-degree factorization*, and *equal-degree factorization*. The first and second algorithms are deterministic, the third is probabilistic.

[van Hoeij] Hoeij, Mark van; Monagan, Michael

“Algorithms for Polynomial GCD Computation over Algebraic Function Fields”

www.cec.m.sfu.ca/personal/mmonagan/papers/AFGCD.pdf

Let L be an algebraic function field in $k \geq 0$ parameters t_1, \dots, t_k . Let f_1, f_2 be non-zero polynomials in $L[x]$. We give two algorithms for computing their gcd. The first, a modular GCD algorithm, is an extension of the modular GCD algorithm for Brown for $\mathbf{Z}[x_1, \dots, x_n]$ and Encarnacion for $\mathbf{Q}(\alpha[x])$ to function fields. The second, a fraction-free algorithm, is a modification of the Moreno Maza

and Rioboo algorithm for computing gcds over triangular sets. The modification reduces coefficient growth in L to be linear. We give an empirical comparison of the two algorithms using implementations in Maple.

[Wang 78] Wang, Paul S.

“An Improved Multivariate Polynomial Factoring Algorithm”

Mathematics of Computation, Vol 32, No 144 Oct 1978, pp1215-1231

www.ams.org/journals/mcom/1978-32-144/S0025-5718-1978-0568284-3/S0025-5718-1978-0568284-3.pdf

A new algorithm for factoring multivariate polynomials over the integers based on an algorithm by Wang and Rothschild is described. The new algorithm has improved strategies for dealing with the known problems of the original algorithm, namely, the leading coefficient problem, the bad-zero problem and the occurrence of extraneous factors. It has an algorithm for correctly predetermining leading coefficients of the factors. A new and efficient p-adic algorithm named EEZ is described. Basically it is a linearly convergent variable-by-variable parallel construction. The improved algorithm is generally faster and requires less store than the original algorithm. Machine examples with comparative timing are included.

[Wiki 4] .

“Polynomial greatest common divisor”

en.wikipedia.org/wiki/Polynomial_greatest_common_divisor

Category Theory

[Baez 09] Baez, John C.; Stay, Mike

“Physics, Topology, Logic and Computation: A Rosetta Stone”

arxiv.org/pdf/0903.0340v3.pdf

In physics, Feynman diagrams are used to reason about quantum processes. In the 1980s, it became clear that underlying these diagrams is a powerful analogy between quantum physics and topology. Namely, a linear operator behaves very much like a “cobordism”: a manifold representing spacetime, going between two manifolds representing space. But this was just the beginning: similar diagrams can be used to reason about logic, where they represent proofs, and computation, where they represent programs. With the rise of interest in quantum cryptography and quantum computation, it became clear that there is an extensive network of analogies between physics, topology, logic and computation. In this expository paper, we make some of these analogies precise using the concept of “closed symmetric monoidal category”. We assume no prior knowledge of category theory, proof theory or computer science.

[Meijer 91] Meijer, Erik; Fokkinga, Maarten; Paterson, Ross

“Functional Programming with Bananas, Lenses, Envelopes and Barbed Wire”

eprints.eemcs.utwente.nl/7281/01/db-utwente-40501F46.pdf

We develop a calculus for lazy functional programming based on recursion operators associated with data type definitions. For these operators we derive various algebraic laws that are useful in deriving and manipulating programs. We shall show that all example functions in Bird and Wadler's "Introduction to Functional Programming" can be expressed using these operators.

[Youssef 04] Youssef, Saul
 "Prospects for Category Theory in Aldor"
 October 2004

Ways of incorporating category theory constructions and results into the Aldor language are discussed. The main features of Aldor which make this possible are identified, examples of categorical constructions are provided and a suggestion is made for a foundation for rigorous results.

Proving Axiom Correct

[Bertot 04] Bertot, Yves; Castéran, Pierre
 "Interactive Theorem Proving and Program Development"
 Springer ISBN 3-540-20854-2

Coq is an interactive proof assistant for the development of mathematical theories and formally certified software. It is based on a theory called the calculus of inductive constructions, a variant of type theory.

This book provides a pragmatic introduction to the development of proofs and certified programs using Coq. With its large collection of examples and exercises it is an invaluable tool for researchers, students, and engineers interested in formal methods and the development of zero-fault software.

[Boulme 00] Boulmé, S.; Hardin, T.; Rioboo, R.
 "Polymorphic Data Types, Objects, Modules and Functors, is it too much?"

Abstraction is a powerful tool for developers and it is offered by numerous features such as polymorphism, classes, modules, and functors, . . . A working programmer may be confused by this abundance. We develop a computer algebra library which is being certified. Reporting this experience made with a language (Ocaml) offering all these features, we argue that they are all needed together. We compare several ways of using classes to represent algebraic concepts, trying to follow as close as possible mathematical specification. Then we show how to combine classes and modules to produce code having very strong typing properties. Currently, this library is made of one hundred units of functional code and behaves faster than analogous ones such as Axiom.

[Boulme 01] Boulmé, S.; Hardin, T.; Hirschkoﬀ, D.; Ménissier-Morain, V.; Rioboo, R.
 "On the way to certify Computer Algebra Systems"
 Calculemus-2001

The FOC project aims at supporting, within a coherent software system, the entire process of mathematical computation, starting with proved theories, ending with certified implementations of algorithms. In this paper, we explain our design requirements for the implementation, using polynomials as a running example. Indeed, proving correctness of implementations depends heavily on the way this design allows mathematical properties to be truly handled at the programming level.

The FOC project, started at the fall of 1997, is aimed to build a programming environment for the development of certified symbolic computation. The working languages are Coq and Ocaml. In this paper, we present first the motivations of the project. We then explain why and how our concern for proving properties of programs has led us to certain implementation choices in Ocaml. This way, the sources express exactly the mathematical dependencies between different structures. This may ease the achievement of proofs.

[Daly 10] Daly, Timothy

“Intel Instruction Semantics Generator”

daly.axiom-developer.org/TimothyDaly_files/publications/sei/intel/intel.pdf

Given an Intel x86 binary, extract the semantics of the instruction stream as Conditional Concurrent Assignments (CCAs). These CCAs represent the semantics of each individual instruction. They can be composed to represent higher level semantics.

[Danielsson 06] Danielsson, Nils Anders; Hughes, John; Jansson, Patrik; Gibbons, Jeremy

“Fast and Loose Reasoning is Morally Correct”

ACM POPL’06 January 2005, Charleston, South Carolina, USA

Functional programmers often reason about programs as if they were written in a total language, expecting the results to carry over to non-total (partial) languages. We justify such reasoning.

Two languages are defined, one total and one partial, with identical syntax. The semantics of the partial language includes partial and infinite values, and all types are lifted, including the function spaces. A partial equivalence relation (PER) is then defined, the domain of which is the total subset of the partial language. For types not containing function spaces the PER relates equal values, and functions are related if they map related values to related values.

It is proved that if two closed terms have the same semantics in the total language, then they have related semantics in the partial language. It is also shown that the PER gives rise to a bicartesian closed category which can be used to reason about values in the domain of the relation.

[Davenport 12] Davenport, James H.; Bradford, Russell; England, Matthew; Wilson, David

“Program Verification in the presence of complex numbers, functions with branch cuts etc.”

arxiv.org/pdf/1212.5417.pdf

In considering the reliability of numerical programs, it is normal to “limit our study to the semantics dealing with numerical precision”. On the other hand, there is a great deal of work on the reliability of programs that essentially ignores the numerics. The thesis of this paper is that there is a class of problems that fall between these two, which could be described as “does the low-level arithmetic implement the high-level mathematics”. Many of these problems arise because mathematics, particularly the mathematics of the complex numbers, is more difficult than expected: for example the complex function \log is not continuous, writing down a program to compute an inverse function is more complicated than just solving an equation, and many algebraic simplification rules are not universally valid.

The good news is that these problems are *theoretically* capable of being solved, and are *practically* close to being solved, but not yet solved, in several real-world examples. However, there is still a long way to go before implementations match the theoretical possibilities.

[Dolzmann 97] Dolzmann, Andreas; Sturm, Thomas

“Guarded Expressions in Practice”

redlog.dolzmann.de/papers/pdf/MIP-9702.pdf

Computer algebra systems typically drop some degenerate cases when evaluating expressions, e.g. x/x becomes 1 dropping the case $x = 0$. We claim that it is feasible in practice to compute also the degenerate cases yielding *guarded expressions*. We work over real closed fields but our ideas about handling guarded expressions can be easily transferred to other situations. Using formulas as guards provides a powerful tool for heuristically reducing the combinatorial explosion of cases: equivalent, redundant, tautological, and contradictory cases can be detected by simplification and quantifier elimination. Our approach allows to simplify the expressions on the basis of simplification knowledge on the logical side. The method described in this paper is implemented in the REDUCE package GUARDIAN, which is freely available on the WWW.

[Dos Reis 11] Dos Reis, Gabriel; Matthews, David; Li, Yue

“Retargeting OpenAxiom to Poly/ML: Towards an Integrated Proof Assistants and Computer Algebra System Framework”

Calculemus (2011) Springer paradise.caltech.edu/~yli/paper/oa-polym1.pdf

This paper presents an ongoing effort to integrate the Axiom family of computer algebra systems with Poly/ML-based proof assistants in the same framework. A long term goal is to make a large set of efficient implementations of algebraic algorithms available to popular proof assistants, and also to bring the power of mechanized formal verification to a family of strongly typed computer algebra systems at a modest cost. Our approach is based on retargeting the code generator of the OpenAxiom compiler to the Poly/ML abstract machine.

[Dunstan 00a] Dunstan, Martin N.

“Adding Larch/Aldor Specifications to Aldor”

We describe a proposal to add Larch-style annotations to the Aldor programming language, based on our PhD research. The annotations are intended to be machine-checkable and may be used for a variety of purposes ranging from compiler optimizations to verification condition (VC) generation. In this report we highlight the options available and describe the changes which would need to be made to the compiler to make use of this technology.

- [Dunstan 98] Dunstan, Martin; Kelsey, Tom; Linton, Steve; Martin, Ursula
 “Lightweight Formal Methods For Computer Algebra Systems”
www.cs.st-andrews.ac.uk/~tom/pub/issac98.pdf

Demonstrates the use of formal methods tools to provide a semantics for the type hierarchy of the Axiom computer algebra system, and a methodology for Aldor program analysis and verification. There are examples of abstract specifications of Axiom primitives.

- [Dunstan 99a] Dunstan, MN
 “Larch/Aldor - A Larch BISL for AXIOM and Aldor”
 PhD Thesis, 1999
www.cs.st-andrews.ac.uk/files/publications/Dun99.php

In this thesis we investigate the use of lightweight formal methods and verification conditions (VCs) to help improve the reliability of components constructed within a computer algebra system. We follow the Larch approach to formal methods and have designed a new behavioural interface specification language (BISL) for use with Aldor: the compiled extension language of Axiom and a fully-featured programming language in its own right. We describe our idea of lightweight formal methods, present a design for a lightweight verification condition generator and review our implementation of a prototype verification condition generator for Larch/Aldor.

- [Dunstan 00] Dunstan, Martin; Kelsey, Tom; Martin, Ursula; Linton, Steve
 “Formal Methods for Extensions to CAS”
 FM 99, Toulouse, France, Sept 20-24, 1999, p1758-1777

We demonstrate the use of formal methods tools to provide a semantics for the type hierarchy of the AXIOM computer algebra system, and a methodology for Aldor program analysis and verification. We give a case study of abstract specifications of AXIOM primitives, and provide an interface between these abstractions and Aldor code.

- [Hardin 13] Hardin, David S.; McClurg, Jedidiah R.; Davis, Jennifer A.
 “Creating Formally Verified Components for Layered Assurance with an LLVM to ACL2 Translator”
www.jrmccclurg.com/papers/law_2013_paper.pdf

This paper describes an effort to create a library of formally verified software component models from code that have been compiled using the Low-Level Virtual Machine (LLVM) intermediate form. The idea is to build a translator from

LLVM to the applicative subset of Common Lisp accepted by the ACL2 theorem prover. They perform verification of the component model using ACL2's automated reasoning capabilities.

- [Hardin 14] Hardin, David S.; Davis, Jennifer A.; Greve, David A.; McClurg, Jedidiah R.
 “Development of a Translator from LLVM to ACL2”
arxiv.org/pdf/1406.1566

In our current work a library of formally verified software components is to be created, and assembled, using the Low-Level Virtual Machine (LLVM) intermediate form, into subsystems whose top-level assurance relies on the assurance of the individual components. We have thus undertaken a project to build a translator from LLVM to the applicative subset of Common Lisp accepted by the ACL2 theorem prover. Our translator produces executable ACL2 formal models, allowing us to both prove theorems about the translated models as well as validate those models by testing. The resulting models can be translated and certified without user intervention, even for code with loops, thanks to the use of the `def::ung` macro which allows us to defer the question of termination. Initial measurements of concrete execution for translated LLVM functions indicate that performance is nearly 2.4 million LLVM instructions per second on a typical laptop computer. In this paper we overview the translation process and illustrate the translator's capabilities by way of a concrete example, including both a functional correctness theorem as well as a validation test for that example.

- [Lamport 02] Lamport, Leslie
 “Specifying Systems”
research.microsoft.com/en-us/um/people/lamport/tla/book-02-08-08.pdf Addison-Wesley ISBN 0-321-14306-X

- [Mason 86] Mason, Ian A.
 “The Semantics of Destructive Lisp”
 Center for the Study of Language and Information ISBN 0-937073-06-7

Our basic premise is that the ability to construct and modify programs will not improve without a new and comprehensive look at the entire programming process. Past theoretical research, say, in the logic of programs, has tended to focus on methods for reasoning about individual programs; little has been done, it seems to us, to develop a sound understanding of the process of programming – the process by which programs evolve in concept and in practice. At present, we lack the means to describe the techniques of program construction and improvement in ways that properly link verification, documentation and adaptability.

- [Newcombe 13] Newcombe, Chris; Rath, Tim; Zhang, Fan; Munteanu, Bogdan; Brooker, Marc; Deardeuff, Michael
 “Use of Formal Methods at Amazon Web Services”
research.microsoft.com/en-us/um/people/lamport/tla/formal-methods-amazon.pdf

In order to find subtle bugs in a system design, it is necessary to have a precise description of that design. There are at least two major benefits to writing a precise

design; the author is forced to think more clearly, which helps eliminate “plausible hand-waving”, and tools can be applied to check for errors in the design, even while it is being written. In contrast, conventional design documents consist of prose, static diagrams, and perhaps pseudo-code in an ad hoc untestable language. Such descriptions are far from precise; they are often ambiguous, or omit critical aspects such as partial failure or the granularity of concurrency (i.e. which constructs are assumed to be atomic). At the other end of the spectrum, the final executable code is unambiguous, but contains an overwhelming amount of detail. We needed to be able to capture the essence of a design in a few hundred lines of precise description. As our designs are unavoidably complex, we need a highly-expressive language, far above the level of code, but with precise semantics. That expressivity must cover real-world concurrency and fault-tolerance. And, as we wish to build services quickly, we wanted a language that is simple to learn and apply, avoiding esoteric concepts. We also very much wanted an existing ecosystem of tools. We found what we were looking for in TLA+, a formal specification language.

- [Poll 99a] Poll, Erik
 “The Type System of Axiom”
www.cs.ru.nl/E.Poll/talks/axiom.pdf

This is a slide deck from a talk on the correspondence between Axiom/Aldor types and Logic.

- [Poll 99] Poll, Erik; Thompson, Simon
 “The Type System of Aldor”
www.cs.kent.ac.uk/pubs/1999/874/content.ps

This paper gives a formal description of – at least a part of – the type system of Aldor, the extension language of the Axiom. In the process of doing this a critique of the design of the system emerges.

- [Poll (a)] Poll, Erik; Thompson, Simon
 “Adding the axioms to Axiom. Toward a system of automated reasoning in Aldor”
citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.7.1457&rep=rep1&type=ps

This paper examines the proposal of using the type system of Axiom to represent a logic, and thus to use the constructions of Axiom to handle the logic and represent proofs and propositions, in the same way as is done in theorem provers based on type theory such as Nuprl or Coq.

The paper shows an interesting way to decorate Axiom with pre- and post-conditions.

The Curry-Howard correspondence used is

PROGRAMMING		LOGIC
Type		Formula
Program		Proof
Product/record type	(..., ...)	Conjunction
Sum/union type	\vee	Disjunction

Function type	->	Implication
Dependent function type	$(x:A) \rightarrow B(x)$	Universal quantifier
Dependent product type	$(x:A, B(x))$	Existential quantifier
Empty type	Exit	Contradictory proposition
One element type	Triv	True proposition

[Poll 00] Poll, Erik; Thompson, Simon

“Integrating Computer Algebra and Reasoning through the Type System of Aldor”

A number of combinations of reasoning and computer algebra systems have been proposed; in this paper we describe another, namely a way to incorporate a logic in the computer algebra system Axiom. We examine the type system of Aldor – the Axiom Library Compiler – and show that with some modifications we can use the dependent types of the system to model a logic, under the Curry-Howard isomorphism. We give a number of example applications of the logic we construct and explain a prototype implementation of a modified type-checking system written in Haskell.

Interval Arithmetic

[Boehm 86] Boehm, Hans-J.; Cartwright, Robert; Riddle, Mark; O'Donnell, Michael J.

“Exact Real Arithmetic: A Case Study in Higher Order Programming”

dev.acm.org/pubs/citations/proceedings/lfp/319838/p162-boehm

[Briggs 04] Briggs, Keith

“Exact real arithmetic”

keithbriggs.info/documents/xr-kent-talk-pp.pdf

[Fateman 94] Fateman, Richard J.; Yan, Tak W.

“Computation with the Extended Rational Numbers and an Application to Interval Arithmetic”

www.cs.berkeley.edu/~fateman/papers/extrat.pdf

Programming languages such as Common Lisp, and virtually every computer algebra system (CAS), support exact arbitrary-precision integer arithmetic as well as exact rational number computation. Several CAS include interval arithmetic directly, but not in the extended form indicated here. We explain why changes to the usual rational number system to include infinity and “not-a-number” may be useful, especially to support robust interval computation. We describe techniques for implementing these changes.

[Lambov 06] Lambov, Branimir

“Interval Arithmetic Using SSE-2”

in Lecture Notes in Computer Science, Springer ISBN 978-3-540-85520-0 (2006) pp102-113

Numerics

- [Lefèvre 06] Lefèvre, Vincent; Stehlé, Damien; Zimmermann, Paul
 “Worst Cases for the Exponential Function in the IEEE-754r decimal64 Format”
 in Lecture Notes in Computer Science, Springer ISBN 978-3-540-85520-0 (2006) pp114-125

We searched for the worst cases for correct rounding of the exponential function in the IEEE 754r decimal64 format, and computed all the bad cases whose distance from a breakpoint (for all rounding modes) is less than 10^{-15} ulp, and we give the worst ones. In particular, the worst case for $|x| \geq 3 \times 10^{-11}$ is

$$\exp(9.407822313572878 \times 10^{-2}) = 1.09864568206633850000000000000000278 \dots$$

This work can be extended to other elementary functions in the decimal64 format and allows the design of reasonably fast routines that will evaluate these functions with correct rounding, at least in some situations.

- [Hamming 62] Hamming R. W.
 “Numerical Methods for Scientists and Engineers”
 Dover (1973) ISBN 0-486-65241-6

Advanced Documentation

- [Bostock 14] Bostock, Mike
 “Visualizing Algorithms”
bost.ocks.org/mike/algorithms

This website hosts various ways of visualizing algorithms. The hope is that these kind of techniques can be applied to Axiom.

- [Leeuwen] van Leeuwen, André M.A.
 “Representation of mathematical object in interactive books”

We present a model for the representation of mathematical objects in structured electronic documents, in a way that allows for interaction with applications such as computer algebra systems and proof checkers. Using a representation that reflects only the intrinsic information of an object, and storing application-dependent information in so-called *application descriptions*, it is shown how the translation from the internal to an external representation and *vice versa* can be achieved. Hereby a formalisation of the concept of *context* is introduced. The proposed scheme allows for a high degree of application integration, e.g., parallel evaluation of subexpressions (by different computer algebra systems), or a proof checker using a computer algebra system to verify an equation involving a symbolic computation.

- [Soiffer 91] Soiffer, Neil Morrell
 “The Design of a User Interface for Computer Algebra Systems”
www.eecs.berkeley.edu/Pubs/TechRpts/1991/CSD-91-626.pdf

This thesis discusses the design and implementation of natural user interfaces for Computer Algebra Systems. Such an interface must not only display expressions generated by the Computer Algebra System in standard mathematical notation, but must also allow easy manipulation and entry of expressions in that notation. The user interface should also assist in understanding of large expressions that are generated by Computer Algebra Systems and should be able to accommodate new notational forms.

- [Victor 11] Victor, Bret
 “Up and Down the Ladder of Abstraction”
worrydream.com/LadderOfAbstraction

This interactive essay presents the ladder of abstraction, a technique for thinking explicitly about these levels, so a designer can move among them consciously and confidently.

- [Victor 12] Victor, Bret
 “Inventing on Principle”
www.youtube.com/watch?v=PUv66718DII

This video raises the level of discussion about human-computer interaction from a technical question to a question of effectively capturing ideas. In particular, this applies well to Axiom’s focus on literate programming.

Differential Equations

- [Abramov 95] Abramov, Sergei A.; Bronstein, Manuel; Petkovsek, Marko
 “On Polynomial Solutions of Linear Operator Equations”
www-sop.inria.fr/cafe/Manuel.Bronstein/publications/mb_papers.html

- [Abramov 01] Abramov, Sergei; Bronstein, Manuel
 “On Solutions of Linear Functional Systems”
www-sop.inria.fr/cafe/Manuel.Bronstein/publications/mb_papers.html

We describe a new direct algorithm for transforming a linear system of recurrences into an equivalent one with nonsingular leading or trailing matrix. Our algorithm, which is an improvement to the EG elimination method, uses only elementary linear algebra operations (ranks, kernels, and determinants) to produce an equation satisfied by the degrees of the solutions with finite support. As a consequence, we can bound and compute the polynomial and rational solutions of very general linear functional systems such as systems of differential or (q -)difference equations.

- [Bronstein 96a] Bronstein, Manuel; Petkovsek, Marko
 “An introduction to pseudo-linear algebra”
 Theoretical Computer Science V157 pp3-33 (1966)
www-sop.inria.fr/cafe/Manuel.Bronstein/publications/mb_papers.html

Pseudo-linear algebra is the study of common properties of linear differential and difference operators. We introduce in this paper its basic objects (pseudo-derivations, skew polynomials, and pseudo-linear operators) and describe several recent algorithms on them, which, when applied in the differential and difference cases, yield algorithms for uncoupling and solving systems of linear differential and difference equations in closed form.

[Bronstein xb] Bronstein, Manuel

“Computer Algebra Algorithms for Linear Ordinary Differential and Difference equations”

www-sop.inria.fr/cafe/Manuel.Bronstein/publications/ecm3.pdf

Galois theory has now produced algorithms for solving linear ordinary differential and difference equations in closed form. In addition, recent algorithmic advances have made those algorithms effective and implementable in computer algebra systems. After introducing the relevant parts of the theory, we describe the latest algorithms for solving such equations.

[Bronstein 94] Bronstein, Manuel

“An improved algorithm for factoring linear ordinary differential operators”

www-sop.inria.fr/cafe/Manuel.Bronstein/publications/mb_papers.html

We describe an efficient algorithm for computing the associated equations appearing in the Beke-Schlesinger factorisation method for linear ordinary differential operators. This algorithm, which is based on elementary operations with sets of integers, can be easily implemented for operators of any order, produces several possible associated equations, of which only the simplest can be selected for solving, and often avoids the degenerate case, where the order of the associated equation is less than in the generic case. We conclude with some fast heuristics that can produce some factorizations while using only linear computations.

[Bronstein 90] Bronstein, Manuel

“On Solutions of Linear Ordinary Differential Equations in their Coefficient Field”

www-sop.inria.fr/cafe/Manuel.Bronstein/publications/mb_papers.html

We describe a rational algorithm for finding the denominator of any solution of a linear ordinary differential equation in its coefficient field. As a consequence, there is now a rational algorithm for finding all such solutions when the coefficients can be built up from the rational functions by finitely many algebraic and primitive adjunctions. This also eliminates one of the computational bottlenecks in algorithms that either factor or search for Liouvillian solutions of such equations with Liouvillian coefficients.

[Bronstein 96] Bronstein, Manuel

“ \sum^{IT} – A strongly-typed embeddable computer algebra library”

www-sop.inria.fr/cafe/Manuel.Bronstein/publications/mb_papers.html

We describe the new computer algebra library \sum^{IT} and its underlying design. The development of \sum^{IT} is motivated by the need to provide highly efficient implementations of key algorithms for linear ordinary differential and (q) -difference

equations to scientific programmers and to computer algebra users, regardless of the programming language or interactive system they use. As such, \sum^{IT} is not a computer algebra system per se, but a library (or substrate) which is designed to be “plugged” with minimal efforts into different types of client applications.

[Bronstein 99a] Bronstein, Manuel

“Solving linear ordinary differential equations over $C(x, e^{\int f(x)dx})$ ”

www-sop.inria.fr/cafe/Manuel.Bronstein/publications/mb_papers.html

We describe a new algorithm for computing the solutions in

$$F = C(x, e^{\int f(x)dx})$$

of linear ordinary differential equations with coefficients in F . Compared to the general algorithm, our algorithm avoids the computation of exponential solutions of equations with coefficients in $C(x)$, as well as the solving of linear differential systems over $C(x)$. Our method is effective and has been implemented.

[Bronstein 00] Bronstein, Manuel

“On Solutions of Linear Ordinary Differential Equations in their Coefficient Field”

www-sop.inria.fr/cafe/Manuel.Bronstein/publications/mb_papers.html

We extend the notion of monomial extensions of differential fields, i.e. simple transcendental extensions in which the polynomials are closed under differentiation, to difference fields. The structure of such extensions provides an algebraic framework for solving generalized linear difference equations with coefficients in such fields. We then describe algorithms for finding the denominator of any solution of those equations in an important subclass of monomial extensions that includes transcendental indefinite sums and products. This reduces the general problem of finding the solutions of such equations in their coefficient fields to bounding their degrees. In the base case, this yields in particular a new algorithm for computing the rational solutions of q -difference equations with polynomial coefficients.

[Bronstein 02] Bronstein, Manuel; Lafaille, Sébastien

“Solutions of linear ordinary differential equations in terms of special functions”

www-sop.inria.fr/cafe/Manuel.Bronstein/publications/issac2002.pdf

We describe a new algorithm for computing special function solutions of the form $y(x) = m(x)F(\eta(x))$ of second order linear ordinary differential equations, where $m(x)$ is an arbitrary Liouvillian function, $\eta(x)$ is an arbitrary rational function, and F satisfies a given second order linear ordinary differential equations. Our algorithm, which is based on finding an appropriate point transformation between the equation defining F and the one to solve, is able to find all rational transformations for a large class of functions F , in particular (but not only) the ${}_0F_1$ and ${}_1F_1$ special functions of mathematical physics, such as Airy, Bessel, Kummer and Whittaker functions. It is also able to identify the values of the parameters entering those special functions, and can be generalized to equations of higher order.

[Bronstein 03] Bronstein, Manuel; Trager, Barry M.

“A Reduction for Regular Differential Systems”

www-sop.inria.fr/cafe/Manuel.Bronstein/publications/mega2003.pdf

We propose a definition of regularity of a linear differential system with coefficients in a monomial extension of a differential field, as well as a global and truly rational (i.e. factorisation-free) iteration that transforms a system with regular finite singularities into an equivalent one with simple finite poles. We then apply our iteration to systems satisfied by bases of algebraic function fields, obtaining algorithms for computing the number of irreducible components and the genus of algebraic curves.

[Bronstein 03a] Bronstein, Manuel; Solé, Patrick

“Linear recurrences with polynomial coefficients”

www-sop.inria.fr/cafe/Manuel.Bronstein/publications/mb_papers.html

We relate sequences generated by recurrences with polynomial coefficients to interleaving and multiplexing of sequences generated by recurrences with constant coefficients. In the special case of finite fields, we show that such sequences are periodic and provide linear complexity estimates for all three constructions.

[Bronstein 05] Bronstein, Manuel; Li, Ziming; Wu, Min

“Picard-Vessiot Extensions for Linear Functional Systems”

www-sop.inria.fr/cafe/Manuel.Bronstein/publications/issac2005.pdf

Picard-Vessiot extensions for ordinary differential and difference equations are well known and are at the core of the associated Galois theories. In this paper, we construct fundamental matrices and Picard-Vessiot extensions for systems of linear partial functional equations having finite linear dimension. We then use those extensions to show that all the solutions of a factor of such a system can be completed to solutions of the original system.

[Von Mohrenschildt 94] Von Mohrenschildt, Martin

“Symbolic Solutions of Discontinuous Differential Equations”

e-collection.library.ethz.ch/eserv/eth:39463/eth-39463-01.pdf

[Von Mohrenschildt 98] Von Mohrenschildt, Martin

“A Normal Form for Function Rings of Piecewise Functions”

J. Symbolic Computation (1998) Vol 26 pp607-619

www.cas.mcmaster.ca/~mohrens/JSC.pdf

Computer algebra systems often have to deal with piecewise continuous functions. These are, for example, the absolute value function, signum, piecewise defined functions but also functions that are the supremum or infimum of two functions. We present a new algebraic approach to these types of problems. This paper presents a normal form for a function ring containing piecewise polynomial functions of an expression. The main result is that this normal form can be used to decide extensional equality of two piecewise functions. Also we define supremum and infimum for piecewise functions; in fact, we show that the function

ring forms a lattice. Additionally, a method to solve equalities and inequalities in this function ring is presented. Finally, we give a “user interface” to the algebraic representation of the piecewise functions.

[Weber 06] Weber, Andreas

“Quantifier Elimination on Real Closed Fields and Differential Equations”

cg.cs.uni-bonn.de/personal-pages/weber/publications/pdf/WeberA/Weber2006a.pdf

This paper surveys some recent applications of quantifier elimination on real closed fields in the context of differential equations. Although polynomial vector fields give rise to solutions involving the exponential and other transcendental functions in general, many questions can be settled within the real closed field without referring to the real exponential field. The technique of quantifier elimination on real closed fields is not only of theoretical interest, but due to recent advances on the algorithmic side including algorithms for the simplification of quantifier-free formulae the method has gained practical applications, e.g. in the context of computing threshold conditions in epidemic modeling.

2.5em0pt

Expression Simplification

[Carette 04] Carette, Jacques

“Understanding Expression Simplification”

www.cas.mcmaster.ca/~carette/publications/simplification.pdf

We give the first formal definition of the concept of *simplification* for general expressions in the context of Computer Algebra Systems. The main mathematical tool is an adaptation of the theory of Minimum Description Length, which is closely related to various theories of complexity, such as Kolmogorov Complexity and Algorithmic Information Theory. In particular, we show how this theory can justify the use of various “magic constants” for deciding between some equivalent representations of an expression, as found in implementations of simplification routines.

Integration

[Baddoura 94] Baddoura, Mohamed Jamil

“Integration in Finite Terms with Elementary Functions and Dilogarithms”

dspace.mit.edu/bitstream/handle/1721.1/26864/30757785.pdf

In this thesis, we report on a new theorem that generalizes Liouville’s theorem on integration in finite terms. The new theorem allows dilogarithms to occur in the integral in addition to elementary functions. The proof is based on two identities for the dilogarithm, that characterize all the possible algebraic relations among dilogarithms of functions that are built up from the rational functions by taking transcendental exponentials, dilogarithms, and logarithms.

- [Bronstein 97] Bronstein, M.
 “Symbolic Integration I—Transcendental Functions.”
 Springer, Heidelberg, 1997 ISBN 3-540-21493-3 evil-wire.org/arrXiv/Mathematics/Bronstein,_Symbolic_Integration_I-Transcendental_Functions.pdf
- [Bronstein 05a] Bronstein, Manuel
 “The Poor Man’s Integrator, a parallel integration heuristic”
www-sop.inria.fr/cafe/Manuel.Bronstein/pmint/pmint.txt
www-sop.inria.fr/cafe/Manuel.Bronstein/pmint/examples
- [Cherry 84] Cherry, G.W.
 “Integration in Finite Terms with Special Functions: The Error Function”
 J. Symbolic Computation (1985) Vol 1 pp283-302
 A decision procedure for integrating a class of transcendental elementary functions in terms of elementary functions and error functions is described. The procedure consists of three mutually exclusive cases. In the first two cases a generalised procedure for completing squares is used to limit the error functions which can appear in the integral of a finite number. This reduces the problem to the solution of a differential equation and we use a result of Risch (1969) to solve it. The third case can be reduced to the determination of what we have termed Σ -decompositions. The result presented here is the key procedure to a more general algorithm which is described fully in Cherry (1983).
- [Cherry 86] Cherry, G.W.
 “Integration in Finite Terms with Special Functions: The Logarithmic Integral”
 SIAM J. Comput. Vol 15 pp1-21 February 1986
- [Cherry 89] Cherry, G.W.
 “An Analysis of the Rational Exponential Integral”
 SIAM J. Computing Vol 18 pp 893-905 (1989)
- [Davenport 79b] Davenport, James Harold
 “On the Integration of Algebraic Functions”
 Springer-Verlag Lecture Notes in Computer Science 102 ISBN 0-387-10290-6
- [Davenport 82] Davenport, J.H.
 “On the Parallel Risch Algorithm (III): Use of Tangents”
 SIGSAM V16 no. 3 pp3-6 August 1982
- [Fateman 02] Fateman, Richard
 “Symbolic Integration”
inst.eecs.berkeley.edu/~cs282/sp02/lects/14.pdf
- [Geddes 92a] Geddes, K.O.; Czapor, S.R.; Labahn, G.
 “The Risch Integration Algorithm”
 Algorithms for Computer Algebra, Ch 12 pp511-573 (1992)
- [Hardy 1916] Hardy, G.H.
 “The Integration of Functions of a Single Variable”
 Cambridge University Press, Cambridge, 1916

[Hermite 1872] Hermite, E.

“Sur l’intégration des fractions rationnelles.”

Nouvelles Annales de Mathématiques (2^{ème} série), 11:145-148, 1872

[Jeffrey 97] Jeffrey, D.J.; Rich, A.D.

“Recursive integration of piecewise-continuous functions”

www.cybertester.com/data/recint.pdf

An algorithm is given for the integration of a class of piecewise-continuous functions. The integration is with respect to a real variable, because the functions considered do not in general allow integration in the complex plane to be defined. The class of integrands includes commonly occurring waveforms, such as square waves, triangular waves, and the floor function; it also includes the signum function. The algorithm can be implemented recursively, and it has the property of ensuring that integrals are continuous on domains of maximum extent.

[Jeffrey 99] Jeffrey, D.J.; Labahn, G.; Mohrenschildt, M.v.; Rich, A.D.

“Integration of the signum, piecewise and related functions”

cs.uwaterloo.ca/~glabahn/Papers/issac99-2.pdf

When a computer algebra system has an assumption facility, it is possible to distinguish between integration problems with respect to a real variable, and those with respect to a complex variable. Here, a class of integration problems is defined in which the integrand consists of compositions of continuous functions and signum functions, and integration is with respect to a real variable. Algorithms are given for evaluating such integrals.

[Knowles 93] Knowles, P.

“Integration of a class of transcendental liouvillian functions with error-functions i”

Journal of Symbolic Computation Vol 13 pp525-543 (1993)

[Knowles 95] Knowles, P.

“Integration of a class of transcendental liouvillian functions with error-functions ii”

Journal of Symbolic Computation Vol 16 pp227-241 (1995)

[Lang 93] Lang, S.

“Algebra”

Addison-Wesley, New York, 3rd edition 1993

[Liouville 1833a] Liouville, Joseph

“Premier mémoire sur la détermination des intégrales dont la valeur est algébrique”

Journal de l'Ecole Polytechnique, 14:124-148, 1833

[Liouville 1833b] Liouville, Joseph

“Second mémoire sur la détermination des intégrales dont la valeur est algébrique”

Journal de l'Ecole Polytechnique, 14:149-193, 1833

[Liouville 1833c] Liouville, Joseph

“Note sur la détermination des intégrales dont la valeur est algébrique”

Journal für die Reine und Angewandte Mathematik, Vol 10 pp 247-259, (1833)

[Liouville 1833d] Liouville, Joseph

“Sur la détermination des intégrales dont la valeur est algébrique”
Journal de l'Ecole Polytechnique, 14:124-193, 1833

[Liouville 1835] Liouville, Joseph

“Mémoire sur l'intégration d'une classe de fonctions transcendentes”
Journal für die Reine und Angewandte Mathematik, Vol 13(2) pp 93-118, (1835)

[Moses 71a] Moses, Joel

“Symbolic Integration: The Stormy Decade”
www-inst.eecs.berkeley.edu/~cs282/sp02/readings/moses-int.pdf

Three approaches to symbolic integration in the 1960's are described. The first, from artificial intelligence, led to Slagle's SAINT and to a large degree to Moses' SIN. The second, from algebraic manipulation, led to Monove's implementation and to Horowitz' and Tobey's reexamination of the Hermite algorithm for integrating rational functions. The third, from mathematics, led to Richardson's proof of the unsolvability of the problem for a class of functions and for Risch's decision procedure for the elementary functions. Generalizations of Risch's algorithm to a class of special functions and programs for solving differential equations and for finding the definite integral are also described.

[Ostrowski 46] Ostrowski, A.

“Sur l'intégrabilité élémentaire de quelques classes d'expressions”
Comm. Math. Helv., Vol 18 pp 283-308, (1946)

[Raab 13] Raab, Clemens G.

“Generalization of Risch's Algorithm to Special Functions”
arxiv.org/pdf/1305.1481.pdf

Symbolic integration deals with the evaluation of integrals in closed form. We present an overview of Risch's algorithm including recent developments. The algorithms discussed are suited for both indefinite and definite integration. They can also be used to compute linear relations among integrals and to find identities for special functions given by parameter integrals. The aim of this presentation is twofold: to introduce the reader to some basic idea of differential algebra in the context of integration and to raise awareness in the physics community of computer algebra algorithms for indefinite and definite integration.

[Raab xx] Raab, Clemens G.

“Integration in finite terms for Liouvillian functions”
www.mmrc.iss.ac.cn/~dart4/posters/Raab.pdf

Computing integrals is a common task in many areas of science, antiderivatives are one way to accomplish this. The problem of integration in finite terms can be stated as follows. Given a differential field (F, D) and $f \in F$, compute g in some elementary extension of (F, D) such that $Dg = f$ if such a g exists.

This problem has been solved for various classes of fields F . For rational functions $(C(x), \frac{d}{dx})$ such a g always exists and algorithms to compute it are known already

for a long time. In 1969 Risch published an algorithm that solves this problem when (F, D) is a transcendental elementary extension of $(C(x), \frac{d}{dx})$. Later this has been extended towards integrands being Liouvillian functions by Singer et. al. via the use of regular log-explicit extensions of $(C(x), \frac{d}{dx})$. Our algorithm extends this to handling transcendental Liouvillian extensions (F, D) of $(C, 0)$ directly without the need to embed them into log-explicit extensions. For example, this means that

$$\int (z - x)x^{z-1}e^{-x}dx = x^ze^{-x}$$

can be computed without including $\log(x)$ in the differential field.

[Risch 68] Risch, Robert

“On the integration of elementary functions which are built up using algebraic operations”

Research Report SP-2801/002/00, System Development Corporation, Santa Monica, CA, USA, 1968

[Risch 69a] Risch, Robert

“Further results on elementary functions”

Research Report RC-2042, IBM Research, Yorktown Heights, NY, USA, 1969

[Risch 69b] Risch, Robert

“The problem of integration in finite terms”

Transactions of the American Mathematical Society 139:167-189, 1969

[Risch 69c] Risch, Robert

“The Solution of the Problem of Integration in Finite Terms”

www.ams.org/journals/bull/1970-76-03/S0002-9904-1970-12454-5/S0002-9904-1970-12454-5.pdf

The problem of integration in finite terms asks for an algorithm for deciding whether an elementary function has an elementary indefinite integral and for finding the integral if it does. “Elementary” is used here to denote those functions build up from the rational functions using only exponentiation, logarithms, trigonometric, inverse trigonometric and algebraic operations. This vaguely worded question has several precise, but inequivalent formulations. The writer has devised an algorithm which solves the classical problem of Liouville. A complete account is planned for a future publication. The present note is intended to indicate some of the ideas and techniques involved.

[Risch 79] Risch, Robert

“Algebraic properties of the elementary functions of analysis”

American Journal of Mathematics, 101:743-759, 1979

[Ritt 48] Ritt, J.F.

“Integration in Finite Terms”

Columbia University Press, New York 1948

- [Rosenlicht 72] Rosenlicht, Maxwell
 “Integration in finite terms”
American Mathematical Monthly, 79:963-972, 1972
- [Rothstein 76] Rosenlicht, Maxwell
 “Aspects of symbolic integration and simplification of exponential and primitive functions”
 PhD thesis, University of Wisconsin-Madison (1976)
- [Rothstein 77] Rothstein, Michael
 “A new algorithm for the integration of exponential and logarithmic functions”
 In *Proceedings of the 1977 MACSYMA Users Conference*, pages 263-274. NASA Pub CP-2012, 1977
- [Seidenberg 58] Seidenberg, Abraham
 “Abstract differential algebra and the analytic case”
Proc. Amer. Math. Soc. Vol 9 pp159-164 (1958)
- [Seidenberg 69] Seidenberg, Abraham
 “Abstract differential algebra and the analytic case. II”
Proc. Amer. Math. Soc. Vol 23 pp689-691 (1969)
- [Singer 85] Singer, M.F.; Saunders, B.D.; Caviness, B.F.
 “An extension of Liouville’s theorem on integration in finite terms”
SIAM J. of Comp. Vol 14 pp965-990 (1985)
- [Trager 76] Trager, Barry
 “Algebraic factoring and rational function integration”
 In *Proceedings of SYMSAC’76* pages 219-226, 1976
- [Trager 84] Trager, Barry
 “On the integration of algebraic functions”
 PhD thesis, MIT, Computer Science, 1984
- [Würfl 07] Würfl, Andreas
 “Basic Concepts of Differential Algebra”
www14.in.tum.de/konferenzen/Jass07/courses/1/Wuerfl/wuerfl_paper.pdf

Modern computer algebra systems symbolically integrate a vast variety of functions. To reveal the underlying structure it is necessary to understand infinite integration not only as an analytical problem but as an algebraic one. Introducing the differential field of elementary functions we sketch the mathematical tools like Liouville’s Principle used in modern algorithms. We present Hermite’s method for integration of rational functions as well as the Rothstein/Trager method for rational and for elementary functions. Further applications of the mentioned algorithms in the field of ODE’s conclude this paper.

Partial Fraction Decomposition

- [Angell] Angell, Tom
 “Guidelines for Partial Fraction Decomposition”
www.math.udel.edu/~angell/partfrac_I.pdf
- [Laval 08] Laval, Philippe B.
 “Partial Fractions Decomposition”
www.math.wisc.edu/~park/Fall2011/integration/Partial%20Fraction.pdf
- [Mudd 14] Harvey Mudd College
 “Partial Fractions”
www.math.hmc.edu/calculus/tutorials/partial_fractions/partial_fractions.pdf
- [Rajasekaran 14] Rajasekaran, Raja
 “Partial Fraction Expansion”
www.utdallas.edu/~raja1/EE4361%20Spring%2014/Lecture%20Notes/Partial%20Fractions.pdf
- [Wootton 14] Wootton, Aaron
 “Integration of Rational Functions by Partial Fractions”
faculty.up.edu/wootton/calc2/section7.4.pdf

Ore Rings

This is used as a reference for the `LeftOreRing` category, in particular, the least left common multiple (`lcmCoef`) function.

- [Delenclos 06] Delenclos, Jonathon; Leroy, André
 “Noncommutative Symmetric functions and W -polynomials”
arxiv.org/pdf/math/0606614.pdf

Let K , S , D be a division ring an endomorphism and a S -derivation of K , respectively. In this setting we introduce generalized noncommutative symmetric functions and obtain Viète formula and decompositions of different operators. W -polynomials show up naturally, their connetions with P -independency. Vandermonde and Wronskian matrices are briefly studied. The different linear factorizations of W -polynomials are analysed. Connections between the existence of LCM (least left common multiples) of monic linear polynomials with coefficients in a ring and the left duo property are established at the end of the paper.

- [Abramov 05] Abramov, S.A.; Le, H.Q.; Li, Z.
 “Univariate Ore Polynomial Rings in Computer Algebra”
www.mmrc.iss.ac.cn/~zmli/papers/oretools.pdf

We present some algorithms related to rings of Ore polynomials (or, briefly, Ore rings) and describe a computer algebra library for basic operations in an arbitrary Ore ring. The library can be used as a basis for various algorithms in Ore rings, in particular, in differential, shift, and q -shift rings.